Higher representations and Heegaard-Floer theory III

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Monoidal category for $\mathfrak{gl}(1|1)^{>0}$

(Joint work with Andrew Manion)

2-rep = diff. cat $/k$, char $k = 2$ $\mathcal{V} \circlearrowleft E$, $\tau \in \mathrm{End}(E^2)$, $d(\tau) = 1$, $\tau^2 = 0$, $E\tau \circ \tau E \circ E\tau = \tau E \circ E\tau \circ \tau E$

$(\mathcal{W}, E_1, \tau_1, E_2, \tau_2, \sigma)$ lax bimod 2-rep. $\sigma: E_2 E_1 \to E_1 E_2$ compatible with τ_1, τ_2
($\mathcal{W}$ closed under cones)

* $\Delta\mathcal{W}$ diff. cat. Obj.: (m, π), $m \in \mathcal{V}$ and $\pi \in \mathrm{Hom}(E_2(m), E_1(m))$, $d(\pi) = 0$
and $\tau_1 \circ (E_1\pi \circ \sigma \circ E_2\pi) = (E_1\pi \circ \sigma \circ E_2\pi) \circ \tau_2$

* $E \circlearrowright \Delta\mathcal{W}$, $E(m, \pi) = \left(\mathrm{Cone}(\pi), \begin{pmatrix} \sigma \circ E_2\pi \circ \tau_2 & \sigma \\ 0 & \tau_1 \circ E_1\pi \circ \sigma \end{pmatrix}\right)$

Assume σ invertible. Define $\tau \in \mathrm{End}(E^2)$, $\tau = \begin{pmatrix} \tau_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma^{-1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_1 \end{pmatrix}$: $\Delta\mathcal{W}$ 2-rep.

Heegaard-Floer theory

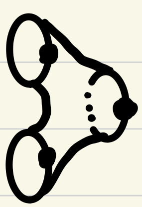
$Y \xrightarrow{X^4} Y' \longmapsto \mathrm{Map}\,\widehat{CFDA}(Y) \to \widehat{CFDA}(Y')$

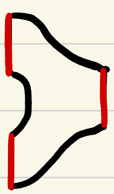
$\Sigma \xrightarrow{Y^3} \Sigma' \longmapsto \widehat{CFDA}(Y)$ $(\mathcal{A}(\Sigma), \mathcal{A}(\Sigma'))$-bimod (Ozsváth-Szabó)

$\Sigma^2 \longmapsto \mathcal{A}(\Sigma)$ diff alg$/\mathbb{F}_2$, $D(\mathcal{A}(\Sigma)) \simeq \mathrm{Fuk}_{\dots}(\mathrm{Sym}^g \Sigma)$ (Lipshitz-Ozsváth-Thurston)

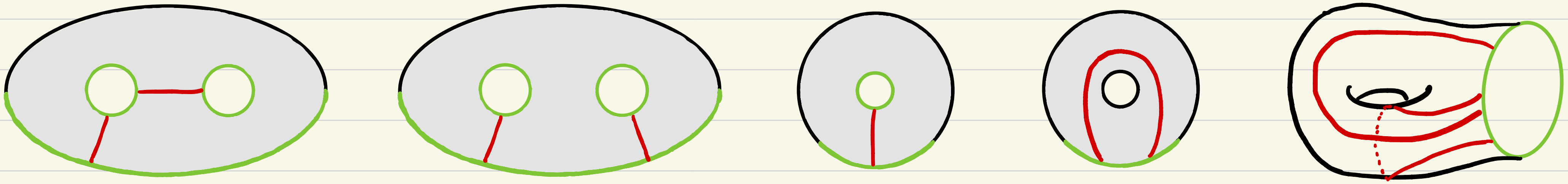
$I^m \xrightarrow{\Sigma^2} I^n \longmapsto (\mathcal{U}^n, \mathcal{U}^m)$-lax bimod $\mathcal{A}(\Sigma)$ (Douglas-Lipshitz-Manolescu, Manion-R)

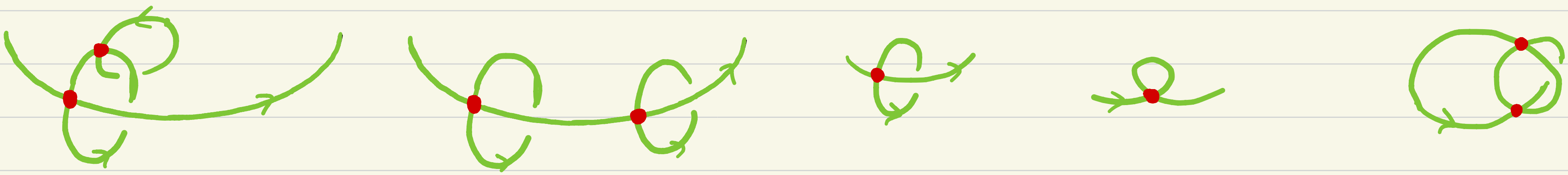
L-O-T: need $\bigcirc \subset \partial\Sigma$. Need also handle dec.

 $\longleftrightarrow \Delta : (\mathcal{U},\mathcal{U})$-bimod $\to \mathcal{U}$-mod

Better:  : partial open-closed 2d TQFT, $| \longmapsto \mathcal{U}$-mod

Surfaces



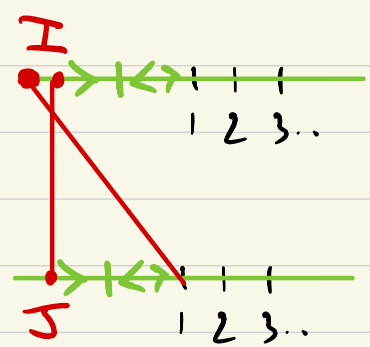


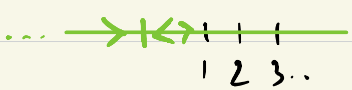
Z singular curve $\longmapsto$ $\mathcal{A}_M(Z)$: obj = fin. subsets of M, Hom = $\mathbb{F}_2\langle$smooth oriented paths$\rangle/\sim$

$\overset{\cup}{M} \supset$ sing. points

$\{$ assoc. graded (filt. from degree)

Lipshitz-Ozsváth-Thurston, Zarev, Manion-R $\rightarrow$ $\mathcal{S}_M(Z)$: differential cat. $d(\,X\,) =)\,($



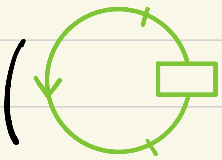
$\cdots \to$ cZ :  : $\tilde{Z}$

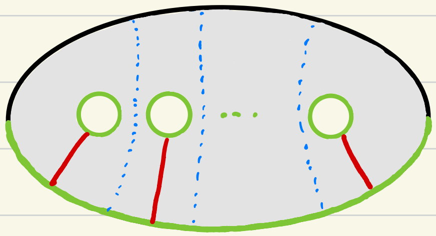
$\rightsquigarrow E^n(I, J) = \mathrm{Hom}_{\mathcal{S}_{M \circ \mathbb{Z}_{>0}}(\tilde{Z})}(I, J \amalg \{1, \dots, n\})$

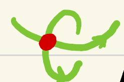
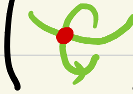
$= 0$

2-rep on $\mathcal{S}_M(Z)$

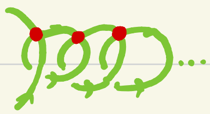
Thm | $\Delta\big(\mathcal{S}($ 

$)\big) \xrightarrow{\sim} \mathcal{S}($  $)$

Ex 

$\mathcal{S}($  $) \simeq \mathcal{S}($  $)^{\otimes n}$, $\mathcal{S}($  $) = \begin{matrix}\mathbb{F}_2[x]\\ \mathbb{F}_2\end{matrix}$ standard basis

?? Derived

$\mathcal{S}($  $)$ H^* is defor. of quot. of $\mathcal{O}^{par}(\mathfrak{gl}_n)$ dual canonical basis
(Lauda-Manion, Manion-Marengon-Willis)