

# GAP XX Kyoto

## Poster Session

Tuesday, April 21, 14:00

Room 110, RIMS

### Baidehi Chattopadhyay

University of Maryland, College Park

*Motivic Hilbert Zeta function of curves*

We give an introduction to the Motivic Hilbert zeta function of curves. We survey recent results on their rationality in various settings and present the (idea of) the proof in the case of reduced curves. We also outline ongoing work aimed at establishing rationality for non-reduced curves with generically reduced singularities at a single point.

### Lorenzo Cortelli

SISSA (Trieste, Italy) and IMPA (Rio De Janeiro, Brazil)

*Deformations of the moduli space of vector bundles over a stacky curve*

In 1975 Narasimhan and Ramanan proved that, under mild numerical conditions, the deformations of the moduli space of (semi)stable vector bundles on a curve are precisely those induced by the deformations of the curve itself. If we now add some stacky points to the curve, i.e. if we take a stacky curve  $C$ , we can still introduce a notion of semistability and get a moduli space of semistable vector bundles which is a projective variety. What are the deformations of this variety? In this poster we go over the steps necessary to generalize Narasimhan and Ramanan's result to the case of vector bundles on stacky curves, we then state a

conjecture and highlight its relevance to the theory of moduli of Fano varieties.

### Timofey Fedorov

HSE University, Russia, Moscow

*Lagrangianity of the Arithmetic and Singular Support of a Holonomic D-Module*

In the theory of D-modules over the field  $\mathbb{C}$ , the classical theorem of Gabber states that for a D-module  $M$  on a variety  $X$ , its singular support  $SS(M)$  is a coisotropic subvariety of  $T^*X$ ; in particular, the singular support of a holonomic  $M$  is Lagrangian. Kontsevich proposed studying  $p$ -supports defined via reduction of D-modules to positive characteristic; the usual (singular) support is then a degeneration of the  $p$ -support. Thomas Bitoun proved the Lagrangianity of  $p$ -supports of holonomic D-modules for all sufficiently large primes  $p$  and deduced the Lagrangianity of the singular support from it. The poster will present Bitoun's result and its connection to the equivalence of the Jacobian and Dixmier conjectures (Tsuchimoto, Kontsevich and Kanel-Belov).

### Taesu Kim

POSTECH

*Categorical structures for Kuranishi spaces*

We introduce a new definition of Kuranishi spaces by associating, to each chart,

$L_\infty[1]$ -algebras defined on open neighborhoods of the zero points of the Kuranishi section. These objects collectively form a category, into which the category of smooth manifolds naturally embeds. The tangent bundle condition for a chart embedding is interpreted as a quasi-isomorphism condition for the  $L_\infty$ -structures. In this process, the originally strict and rigid cocycle condition for coordinate changes is replaced by more flexible homotopy-theoretic compatibilities. To this end, a model of higher homotopy theory for  $L_\infty[1]$ -morphisms is proposed. The moduli space of pseudoholomorphic disks with Lagrangian boundary condition can be shown to serve as an example, provided that the closed two-form allows a 'nice' stratification on each chart. In particular, the forgetful and evaluation maps for the moduli space are also lifted to morphisms in our category.

### David Klompenhouwer

University of Padova

*Meromorphic differentials, spin structures, and the BKP hierarchy*

A meromorphic differential on an algebraic curve with even orders at zeros and poles induces a spin structure on the curve. The strata of curves that admit such differentials and spin structures behave reasonably well with respect to the boundary maps of the moduli space of stable curves, i.e. they form a partial Cohomological Field Theory (CohFT). We show that the intersection-theoretic properties of this partial CohFT are encoded by the BKP hierarchy, a system of PDEs which is a reduction of the well-known KP hierarchy.

### Alex Villaro Krüger

Higher School of Economics, Steklov Mathematical Institute

*Fano Threefolds of Type 4-1*

A Fano Threefold of type 4-1 is a smooth divisor in  $(P^1)^4$  of multidegree  $(1, 1, 1, 1)$ .

The Hilbert scheme of twisted quartic curves on such a threefold is an isotrivial fibration over the intermediate Jacobian with all fibers isomorphic to the original threefold. Using this it is possible to classify all possible automorphism groups, and show that they are semi-direct products of  $Z_2^4$  by the group of automorphisms of a genus-one curve fixing a degree-3 divisor. The moduli space is 3-dimensional and it is possible to show which threefolds have which automorphism groups on this moduli space. Finally, the semiorthogonal decomposition has a component which is equivalent to the derived category of a genus-one curve with 3 stacky points. For forms of such threefolds, over non-closed fields with Picard rank 1, this category is still defined, but the stacky curve is not, so it looks like a non-commutative (maybe Brauer twisted) form of such a stacky curve.

### Shengxuan Liu

University of Michigan

*Inducing t-structures on semiorthogonal components*

I will discuss a new method for constructing t-structures on semiorthogonal components of triangulated categories, which leads in particular to the first examples of bounded t-structures on phantom categories. This is joint work with Alexander Kuznetsov and Alexander Perry.

### Zihang Liu

The University of Hong Kong

*Exotic Poisson deformations via Belavin-Drinfeld data*

The standard Poisson structures on Schubert cells can be recovered by deforming their log-canonical parts according to a recent theory of Lu and Matviichuk. Motivated by similar considerations for Belavin-Drinfeld (BD) Poisson structures on  $GL_n$ , we use BD data to twist the log-canonical

parts of the standard Poisson structures on Schubert cells and study a class of their Poisson deformations which we call exotic Poisson deformations.

It is conjectured that such deformed Poisson structures recover many known examples, as well as generating far more new ones. It is our ongoing project to extend the GSV program, by showing that the exotic Poisson deformations are compatible with new exotic cluster structures. This is based on joint work with J.-H. Lu and D. Voloshyn.

### **Yuji Okitani**

UC Berkeley

#### *Functoriality in 3d Mirror Symmetry*

Gammage, Hilburn and Mazel-Gee formulated 3d mirror symmetry as an equivalence of 2-categories, where the A-side is modelled by perverse schobers (or more precisely spherical adjunctions) and the B-side is modelled by coherent categories of sheaves on  $A1/Gm$ , denoted  $2Coh(A1/Gm)$ . I outline two instances of functoriality in this equivalence.

The first instance is joint work with Peng Zhou. We construct a monoidal structure on  $2Coh(A1/Gm)$  by viewing  $A1/Gm$  as a monoidal stack and show that under mirror symmetry this is a categorification of the Thom-Sebastiani formula. We show furthermore an interesting failure of “Knorrer periodicity” and apply this in ongoing work to develop a microlocal theory of perverse schobers.

Another instance is joint work with Swapnil Garg and Ruoxi Li. Bodzenta and Donovan study the derived category of a root stack and use window theory to prove that this admits a certain periodic semi-orthogonal decomposition. We reinterpret their work as the study of certain objects and operations on  $2Coh(A1/Gm)$ , and compute their mirrors. We apply this to give a different proof of their results which doesn't

use window theory.

### **Kaichuan Qi**

Penn State University

#### *Construction of $\mathbb{S}^1$ -Gerbes over the Stack $[G/G]$*

We give an explicit construction of  $\mathbb{S}^1$ -gerbes over the differentiable stack  $[G/G]$ , where  $G$  is a compact and connected Lie group. Our construction provides a complete and detailed realization of results previously announced by Behrend-Xu-Zhang, using the description of gerbes over stacks as  $\mathbb{S}^1$ -central extensions of Lie groupoids. Moreover, we prove that the Dixmier-Douady class of the resulting gerbe coincides with the Alekseev-Malkin-Meinrenken equivariant 3-class; under the additional assumption that  $G$  is compact, simple, and simply connected, this class represents the generator of  $H_G^3(G, \mathbb{Z})$ .

### **Maksim Retinskii**

HSE University

#### *Around the noncommutative Cartan homotopy*

The classical Cartan homotopy formula is a fundamental and powerful tool in algebraic and differential geometry. A natural and significant challenge is to extend this formula to the non-commutative setting.

While it is known that the non-commutative version of Lie derivative acts trivially on cyclic homology — a non-commutative analogue of de Rham cohomology — an explicit homotopy formula, analogous to the commutative case, has remained elusive. This absence obstructs the development of a genuine non-commutative Hodge theory, particularly the construction of a real structure required for an R-Hodge structure.

In this speech, we will present a new construction that (we hope) could generalize the Cartan homotopy formula to the non-commutative context. This framework not

only provides the missing explicit formula but also allow us to find more explicit connection between non-commutative and de Rham world.

### Samson Saneblidze

Iv. Javakhishvili Tbilisi State University

*Combinatorial classifying and its dual spaces*

For a monoidal cubical set  $M$  we functorially construct the simplicial set  $\mathbf{B}M$  such that  $\mathbf{\Omega}M$  is identified with  $M$  whenever it is free as monoid and the face operator  $d_*^0$  in  $M$  is quadratic, where  $\mathbf{\Omega}$  is the functor from the category of simplicial sets to the category of monoidal cubical sets. As an application we immediately establish the localization isomorphism  $H_*(|M|)[\pi^{-1}] \approx H_*(\mathbf{\Omega}B|M|)$ , where  $|M|$  is the geometric realization of  $M$ ,  $\pi := \pi_0(|M|)$  is the center of the homology  $H_*(|M|)$ ,  $B|M|$  is the standard classifying space and  $\mathbf{\Omega}$  is the based loops on  $B|M|$ .

### Alexandra Sonina

Steklov Mathematical Institute of Russian Academy of Sciences

*Finite Subgroups of Automorphism Groups of Non-trivial Severi–Brauer Varieties*

A Severi–Brauer variety over a field  $K$  is an algebraic variety that becomes isomorphic to projective space over an algebraic closure of  $K$ . Severi–Brauer varieties demonstrate how much richer and more diverse algebraic geometry over non-algebraically closed fields is than geometry over algebraically closed fields. We classify finite subgroups of automorphism groups of non-trivial Severi–Brauer varieties of dimension  $p - 1$ , where  $p \geq 3$  is prime, over an arbitrary field whose characteristic is coprime to  $p$ . In addition, we construct families of examples i.e., for every consistent set of finite groups we construct a field together with a non-trivial Severi–Brauer varieties over that

field such that every group in this set of finite subgroups acts on the constructed variety.

### Sara Stephens

Cornell University

*Wall-Crossing for Quasimaps to Projective Space*

Moduli spaces of  $\epsilon$ -stable quasimaps, introduced by Ciocan-Fontanine, Kim, and Maulik, exhibit a wall-and-chamber structure that interpolates between stable quasimaps and Kontsevich’s stable maps. In this poster, I will describe an intrinsic perspective on the resulting wall-crossing phenomena via algebraic stacks. We construct an algebraic stack encoding an epsilon-stable quasimap wall crossing in the case where the target is projective space, and analyze necessary and sufficient filling conditions for this stack to admit a good moduli space. Varying the Theta-stratification of this algebraic stack leads to a K-theoretic wall-crossing formula for  $\epsilon$ -stable quasimaps.

### Greg Weiler

University of Göttingen

*Lie Algebra Cohomology and Root Lattice Combinatorics: From Borel’s Model to the Nilpotent Cone*

This poster explores the cohomology of the flag manifold  $G/B$  via the relative Chevalley-Eilenberg complex of invariant differential forms. By utilizing Cartan decompositions and complexified weight spaces, we represent the cochains combinatorially as closed paths, or ”loops”, in the root lattice. We provide an explicit cochain-level reconstruction of Borel’s coinvariant model  $H^*(K/T, \mathbb{C}) \cong S(\mathfrak{h}^\vee)/I_W$  using the universal machinery of the Weil algebra  $W\mathfrak{g}$ . Specifically, we demonstrate that Chern-Simons transgression cochains restrict to highly symmetric sums of root loops.

Finally, we outline the primary motivation for developing this dictionary. Our aim is to study the intersection cohomology of stratified spaces, in particular the nilpotent cone, by algebraic means. We hope to exploit the Springer resolution in addition to the finite-dimensional combinatoric methods outlined above to study the extensions of forms living on the regular orbit to the singular boundary. In the spirit of  $L^2$ -cohomology, we hope to develop an algebraic Hilbert-Mumford criterion for this purpose.